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Towards a Mathematical Model of Government Behaviour

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With 7 Figures

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I. Introduction

A. Economics and the Government

Although the government is undoubtedly of overriding importance for the *economy*, it has been mostly treated as an exogenous factor, about which nothing can be said, both in literary and formal economic analysis[1].

In the quantitative approach to economic policy-making[2], a postulated social welfare function is maximised subject to the limitations imposed by the estimated parameters of an econometric model. This yields the optimal values of the policy instruments. The literary approach is not essentially different, except that "intuition" and "common sense" are substituted for the social welfare function to derive the policies necessary to reach the goals of the economy. As the social welfare function has not so far been determined "scientifically"[3], the underlying goals on which all

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[1] There have in the past always been some economic theoreticians who have endeavoured to integrate the government into economics in a way going beyond pure description or normative prescription. The contributions of H. Hotelling (1929) and J. Schumpeter (1942) must be especially mentioned here. These attempts have, however, had practically no impact on contemporary economic theorizing and model building.

[2] See the classic works by J. Tinbergen (1956) and H. Theil (1961).

[3] Very recently some serious attempts have been made to determine the weights of a welfare function. They are unsatisfactory because they only measure

the recommended actions hinge are in both approaches more or less well-informed guesses on the part of the economists.

One of the main shortcomings of much policy advice is that though it supposedly serves to increase the total welfare of the society as perceived by the government[4], it often has no chance of being implemented because it does not take into account the political consequences. It may well be that a policy which seems reasonable from the purely economic point of view leads to a loss of votes for the government undertaking such a policy. The proposals made by the advisors are hence either not followed at all or are thoroughly changed in the political process. The goals as the economic advisors conceive them may therefore mostly be unattainable or, if partially attained, detrimental to the project, as these proposals emerge from the political debate. Thus, even if the advisors' recommendations are optimal *ex ante*, viewed *ex post* they may lead to very unsatisfactory outcomes.

This situation may be improved by a knowledge of government behaviour. It is then possible for the economist to suggest only those policies which have a good chance of being implemented without significant changes. Such "second best" policies (from the point of view of maximising social welfare) may prove to further social welfare more than straightforward "first best" policies which are never put into practice.

The exogeneous nature of the government in contemporary economic analysis also hampers economic forecasting. It is unsatisfactory that there is no theoretical basis to make predictions about approximately one quarter of national income; the careful prediction of the other components may be useless if the assumptions about the government's actions are based on pure speculation. The forecaster must make some assumption of how conflicts amongst various goals (e. g. the conflict between unemployment and inflation) will be solved. What is needed is a behavioural equation for the government describing how it responds to changes in the economic variables under various circumstances. Such forecasts when applied to the other countries will provide much more realistic predictions of international transactions (including trade and capital flows). If it is the government which requires the forecast, it in fact wants a prediction of its own behaviour, given the exogenous circumstances[5].

the subjective welfare function of the decision-makers which may deviate considerably from the "true" social welfare. The following studies are relevant: Reuber (1964), Dewald and Johnson (1963), Wood (1967), Deprano and Nugent (1967).

[4] Note that the social welfare function here is the one perceived by the government. It may be different from the social welfare function as perceived by the population. In particular, it may be very different from the true social welfare function, *if such a function exists*.

[5] See G. Bombach (1962).

B. Political Science and the Government

Both political science and political sociology have in the past dealt with the government mostly in descriptive terms, i. e. without clear-cut behavioural assumptions and derived testable hypotheses, with a few exceptions[6]. Much collection of data and description of actual government behaviour, though painstaking in details, has remained incoherent and only interesting for the specific cases covered, because there has been no theory on which an analysis can proceed.

II. The Contribution of the Economic Theory of Politics

Professor Arrow's pathbreaking book "Social Choice and Individual Values" (1951) has aroused the interest of theoretical economists in the problem of voting from a formal standpoint. There are relatively few attempts to deal with other, more general aspects of the political system. It is A. Downs's "Economic Theory of Democracy" (1957) which has first brought forward a clear-cut behavioural hypothesis for the government and traced through some of the consequences.

A. Downs's Behavioural Assumption

Downs advances the hypothesis that "the main goal of every party is the winning of elections. Thus all its actions are aimed at maximising votes, and it treats policies merely as means towards this end" (1957, p. 35). To any economist it is obvious that this behavioural hypothesis is analogous to the profit maximisation hypothesis for the firm. It is one of the many instances in which there is a close correspondence between the economic and the political system. Downs shows, and later more formal analyses confirm, that it is also possible to derive interesting results from the simple vote maximisation hypothesis[7].

The rather crude hypothesis of straightforward vote maximisation can be modified to correspond better to the specific features of the political system, without changing its basic orientation. The following more realistic variants can be advanced.

B. Maximisation of the Vote Share

The fate of the government and every party in a democracy does not depend on how many votes it receives but rather on its *percentage* of the total vote. This behavioural hypothesis may be different from the maximisation of the number of votes, because the voter participation rate

[6] The so called "behavioural revolution" which attempts to reorientate political science along this line is at present entering slowly into more progressive political science departments. Some of the main advocates are: H. Eulau (1962), R. A. Dahl (1961). For the actual development of such theories compare W. H. Riker (1962) and G. H. Kramer (1966).

[7] See O. A. Davis and M. Hinich (1966 and 1967).

may be influenced. It may thus be disadvantageous for the government to propose a policy which, though it may increase the total number of its votes, leads to an even stronger rise in the number of votes going to the opposition.

C. Maximisation of the Vote Majority

This behavioural hypothesis is similar, but not identical, to vote share maximisation. Under this assumption the government is indifferent between policies leading to an increase of its own vote or an equal decrease of the vote going to the opposition[8].

D. Minimum Winning Majority

It is not necessary for the government to have a majority larger than strictly required, e. g. in the case of a two-party system larger than fifty percent of votes. If the winning majority is able to get some special advantages exclusively for its own partisans, each one's share of the spoils decreases if there are more members in the coalition than strictly necessary[9]. There is another reason why the government may attempt to gain only a minimum majority. When the majority is small, the government leaders can exercise more control over the members of parliament belonging to their party, while a larger majority tends to relax party discipline, because everybody knows that any single parliamentarian is not decisive[10].

E. Maintenance of a Majority

Such a majority combines the advantages of an undisputed parliamentary majority with a good party discipline which makes life easy for the government. In order to have an operational concept, the two opposing forces must, however, be specified more precisely with theoretical arguments. The size of a comfortable majority may be quite different under various institutional conditions.

F. Maximisation of the Probability of Winning the Election

This behavioural hypothesis is slightly different from those mentioned above. It is explicitly formulated in a stochastic context. As the approach used here is deterministic, this formulation is not further considered. It should, however, be borne in mind that a policy which maximises the expected value of the vote share at every election needs not coincide with the policy which maximises the probability of winning at every election.

[8] This hypothesis is used by G. Kramer (1966) and by B. Frey (1968).

[9] Such a game theoretic approach is used by W. H. Riker (1962) to develop a theory of coalition formation.

[10] This seems to be particularly true in the British parliamentary system. It reminds of T. Schelling's (1960) proposition that it is sometimes advantageous to be weak because then the other "players" know that the weaker "player" cannot make any concessions.

III. Internal and External Pressures on Government Behaviour

A. Shortcoming of Present Hypotheses

The Economic Theory of Politics has made a major step forward in the formulation of a useful theory of the government. Its concentration on voting identifies one of the chief factors determining the government's actions.

The emphasis on voting success as the criterion for government behaviour has, however, gone too far. Even casual observations of the political scene shows that another factor, namely the government party's *ideology*, is also of considerable importance.

It would be mistaken to say that the new political economists do not take ideology into account. In fact, a major part of Downs's book deals with the role of ideology in policy formation (chapters 7, 8). Following H. Hotelling (1929) and A. Smithies (1941), he uses the distribution of voters along the political scale to explain the location of ideologies. Ideology is thus seen as the dependent variable, and the preferences of the voters as the independent variable.

This ingenious application of the theory of spatial competition to politics has its merits, but it is evident that ideologies adjust only slowly and the theory thus only applies in the long run. Most decisions of the government are, however, short or intermediate-run, because elections are held in most democracies at regular intervals of between three and five years. If one tries to explain government behaviour over this interval, ideology has an independent influence on government actions which cannot be reduced to voting alone[11].

There is no need to elaborate that the obverse behavioural hypothesis, namely, that governments undertake whatever policies dictated by their ideological preferences, is patently unrealistic. As will be shown subsequently, in a democracy such a government can stay in power only under special circumstances. In most instances, however, it will lose the subsequent elections, and once in the opposition, the party will have no opportunity to change the "world" according to its own ideology.

B. A General Hypothesis of Government Behaviour

The behavioural hypothesis suggested in this paper is more general than either vote maximisation or the maximisation of ideological satisfaction. It is stipulated that the *government endeavours to maximise its*

[11] In his "Model of Economic and Political Decision-Making", J. Rothenberg (1965) takes into account that politicians are not solely concerned about votes, but also enter political life in order to put ideas into practice. If it turns out that the voters reject these ideas or ideologies, the particular politician must decide whether it is still worthwhile to stay in political life or whether the opportunity costs of an occupation in the business sector are getting too high. If he has to modify his ideas too much in order to have any practical success, he may leave whatever political post he occupies.

utility U over all future periods of time. Formally

$$\max U = \int_0^\infty F(t)\, a(t)\, dt, \tag{1}$$

where the integral is taken from $t = 0$ to infinity. $F(t)$ is the instantaneous utility flow at time t. Following Gorman (1957) it is called "felicity". As can be expected, the government (like individuals) values future less than present utility, hence the felicity at any moment is multiplied by the discount factor $a(t)$, a decreasing function of time[12].

The government is seen to perform this dynamic maximisation under pressures from a great many different groups and individuals. Its actions are hence influenced by a number of constraints, which can be viewed as essentially of two kinds, namely, *internal pressures* and *external pressures.*

(a) Internal Pressures on Government Behaviour

Internal pressures on the government are exercised by all those forces which arise from within the government as the decision-making unit. The government is not one solid unit with a single will but rather a conglomeration of subunits such as the chief executive, the cabinet, the party leaders, the party rank and file, etc. All of these have specific interests of their own but at the same time they share a common interest to stay in power. They thus form a coalition with the typical mixture of converging and diverging interests.

Internal pressures are characterised by the fact that the government loses its power not because the voters defeat it in an election, but rather because it breaks up due to internal inconsistencies. This, for instance, may take the form of the resignation of an important member of the government leaving it or the breaking-off of a wing of the party. In a coalition government, one of the parties may leave the coalition. It can be seen that in all these cases the *direct* reason for the government's loss of power is not a defeat in the general elections. In fact, the electorate may confirm or reverse the outcome later on.

There may, however, be an internal breakdown of the government because a group of parliamentarians feel that insufficient attention is paid to voters' opinion. Being aware that such policies may render their *own* reelection impossible, they will attempt to influence the government leaders to conform more to voters' wishes. If the leaders do not respond, they may disassociate themselves from the government, if only to increase the chances of their own reelection. These actions make a change of government likely.

[12] It is customary to assume that at a constant rate of felicity, the utility integral converges. This implies that the integral over time of the discount factor must be finite, i. e.

$$\int_0^\infty a(t)\, d(t) < \infty.$$

Thus, one may hypothesise that the government will attempt to minimise these internal pressures arising from ideology and from popular approval. There exists a tradeoff between ideological satisfaction and popular approval, and in particular, an optimal combination of both. The basic behavioural assumption in this paper is that the government maximises the utility integral (1) over time, with felicity a function of ideological satisfaction I and instantaneous approval A $(0 \leqq A \leqq 1)$. The felicity function is thus written as

$$F = F(I, A, t). \tag{2}$$

A third argument, t, is included to indicate that there may be an exogeneous shift in the felicity function over time. It can be assumed that the first two arguments show positive but decreasing marginal effects on felicity

$$\partial F/\partial I \geqq 0 \qquad \partial^2 F/\partial I^2 \leqq 0, \tag{2a}$$

$$\partial F/\partial A \geqq 0 \qquad \partial^2 F/\partial A^2 \leqq 0. \tag{2b}$$

The felicity function includes vote maximisation and the maximisation of ideological satisfaction as special cases[13].

(b) External Pressures on Government Behaviour

External pressures upon the government are characterised by the fact that they lead to *election defeat* if neglected. The conditions for reelection are well defined: the government must reach a majority in each upcoming election time $T_1, T_2, \ldots T_N$. (The possibility of coalition governments is for the sake of simplicity disregarded). If the election dates are prescribed by the constitution at regular intervals (e. g. every fourth year), the government must acquire a majority at each of the election dates $T, 2T, \ldots NT$. This will be assumed in the following, but can easily be relaxed without altering the conclusions.

It is postulated that success at election time T depends on how much *good-will* the government has been able to accumulate in the past. If the government has consistently been popular with the voters, i. e. if $A(t)$ has always been at a high percentage, it can expect to win the elections. It is natural to assume that the further in the past a government action is taken, the less weight it carries in the voters' decision, other things being equal. An alternative possibility would be that the voters remember all past government actions equally well, but only over a finite period.

[13] Pure maximisation of ideological satisfaction may take place in a totalitarian society. Even in that case, however, there may be increasing costs involved in the suppression of discontent such that unconstrained maximisation of ideological satisfaction may be unsustainable if the felicity function is basically incompatible with the approval function. The ideological satisfaction function may be interpreted as the social welfare function *as perceived by the government*.

In general, the importance attached to a given government action taken at time z by a voter at time t will be represented by

$$[1 - \int_0^{t-z} m(t')\, dt'],$$

where $m(t')$ is the mortality density function[14].

The good-will which the government has accumulated at any given moment of time is defined as the depreciated sum of past instantaneous approvals

$$\pi(t) = \int_{-t_\infty}^{t} A(t')\, dt' - \int_{-t_\infty}^{t} \int_{-t_\infty}^{t'} A(z)\, m(t'-z)\, dz\, dt' + \pi_{-t_\infty}. \tag{3}$$

The first term sums up the approval enjoyed by the government from some distant time in the past $(-t_\infty)$ to the present. The second term indicates how much of each "vintage" of approval has disappeared due to depreciation. The last term gives the good-will of the government of the initial period; it is assumed to be zero.

Differentiation of (3) yields the change in good-will at any given moment of time

$$\dot{\pi}(t) = A(t) - \int_{-t_\infty}^{t} A(z)\, m(t-z)\, dz, \tag{4}$$

$m(t-z)\, dz$ represents the fraction of approval generated during the small interval dz around the time point z that is forgotten by the voters at time t. Alternatively, it can be interpreted as the probability distribution of time that the voters will take into account a given action at z when evaluating the past actions of the government.

Equ. (4) will be of particular importance in the subsequent analysis, as it represents the *transition* equation of a Pontryagin type optimisation problem.

In a democracy, the government will be reelected if it receives a "sufficiently large" share of votes $\overline{V}$. The size of $\overline{V}$ depends on institutional factors such as the specific election rules and the number of competing parties. For any election at NT, N an integer, the actual vote

[14] The mortality density function is commonly employed in the context of capital theory but appears appropriate in this application. Amongst its properties are the following:

$$m(-0) = 0; \tag{i}$$

$$1 \geqq m(t) \geqq 0, \text{ for all } t; \tag{ii}$$

$$\int_0^\infty m(t)\, dt = 1. \tag{iii}$$

The act of forgetting may be compared to the depreciation of capital.

share received by the government $V(NT)$ must be larger than (or at least equal to) $\bar{V}$:[15]

$$V(NT) \geqq \bar{V}. \tag{5}$$

The actual vote received at time t is assumed equal to the "average" good-will enjoyed by the government over the whole past (appropriately depreciated)

$$V(NT) = \frac{\pi(NT)}{\int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'')\, dt'']\, dt'}. \tag{6}$$

The expression in the denominator is a weight, which takes into account forgotten good-will, and which therefore attaches relatively greater importance to the more recent past and it also represents the maximum good-will attainable at time t. If the government has over the past always had the same percentage of voters supporting it, the vote share at election NT turns out to be equal to this constant approval rate, i.e. if

$$A(t') = \bar{A}, \text{ for all } -t_\infty < t' < t$$

then

$$V(NT) = \frac{\pi(NT)}{\int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'')\, dt'']\, dt'} = \bar{A}. \tag{7}$$

The expression for $V(NT)$ also holds for $t = 0$

$$V(0) = \frac{\pi(0)}{\int_{-t_\infty}^{0} [1 - \int_0^{t'+t_\infty} m(t'')\, dt'']\, dt'}. \tag{8}$$

[15] It is possible to allow for the case that the government not only discounts future felicities derived from being in office, but also takes future election requirements less and less seriously. This can be expressed by applying another discount factor $\beta(t)$ a monotonically decreasing function of time, which then gives the following conditions for reelection at subsequent election dates:

$$V(T) \geqq \bar{V}\beta(T)$$
$$V(2T) \geqq \bar{V}\beta(2T)$$
$$V(NT) \geqq \bar{V}\beta(NT).$$

This implies that a vote share of, say, 30 % in an election K terms ahead may from the point of view of today be considered sufficient, even though the government gets reelected only if it receives 50 % of the vote. As this does not substantially alter the argument of the paper, it will be assumed that the reelection requirements are not discounted.

Combining relations (5) to (8) gives

$$\frac{\pi(NT)}{\int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'')\, dt'']\, dt'} \geqq \bar{V},$$

$$\pi(NT) \geqq \bar{V} \left\{ \int_{-t_\infty}^{NT} [1 - \int_0^{t'+t_\infty} m(t'')\, dt'']\, dt' \right\},$$

$$\pi(NT) \geqq \bar{V} \left\{ \frac{\pi(0)}{V(0)} + \int_0^{NT} [1 - \int_0^{t'+t_\infty} m(t'')\, dt'']\, dt' \right\}. \tag{9}$$

This last expression states that in order to be elected, the government must at each election date have accumulated at least the amount of good-will indicated at the right hand side of (9). If it does not succeed in reaching that limit, it will be defeated in this particular election, i. e. it will not reach the necessary vote share $\bar{V}$.[16]

IV. Goals and the Economic System

The government's behaviour is not only limited by political factors but also by the resources available to the society. This means that there are necessarily conflicts to the achievement of the ends. Though not all such conflicts are economic in the narrow sense, the great majority of politically relevant issues are either directly or indirectly of this nature[17].

The politically relevant issues $y_1, y_2, \ldots y_n$ influence both the voting behaviour of the electorate (i. e. appear as arguments in the approval function) and the ideological satisfaction of the government. If only strictly economic goals are considered, they may be composed of the five "traditional" goals of an economy, namely: Price Stability, Adequate Economic Growth, Equilibrium in the Balance of Payments, "Just" Income Distribution and Full Employment. It may be argued that some of these goals are not final but rather intermediate (such as Price Stability); that some are constraints rather than goals (e. g. the Balance of Payments Equilibrium). Moreover, this list does not even cover other

[16] In this paper, it is assumed that the government always wants to be reelected. This must be so as otherwise the utility integral must be reinterpreted; positive felicity can accrue to the government only if it is in office. This particular feature of the model may be modified to conform better to reality by changing the hypothesis to one of maximising the expected value of the utility integral. Then the constraints as indicated by (9) need not hold strictly and will be replaced by probabilistic statements based on accumulated good-will. The resulting policy will become a great deal more flexible. Alternatively, Professor K. J. Arrow suggests a dynamic game formulation with two parties in which a party is explicitly allowed to be intermittently out of office.

[17] If economics is defined as the science dealing with the optimal allocation of scarce resources between competing ends, all conflicts are "economic".

important economic goals such as maximum consumption per head. However, no normative discussion of what the final goals should be is intended here.

Analytically, the goals can be treated in two different ways:

(i) It can be assumed that there is for each goal or political issue an optimal level represented by the vector y_i^* (different for each individual and for each party). The actual performance of the government as expressed by the levels of each goal actually achieved, y, can be compared to these optimal levels. A quadratic loss function may be specified

$$L_i(y) = (y - y_i^*)' M (y - y_i^*),$$

where the subscript i refers to the individual or party. M is a positive definite matrix giving the relative weights of the various issues. It can be shown that this *loss function* can be derived from a quadratic utility function. The latter is widely used in the theory of economic policy because of its desirable economic and mathematical properties[18]. The shortcomings are that positive and negative deviations from the most desired value are treated equally, e. g. a balance of payments surplus is equally undesirable as a deficit. This barely corresponds to reality. Moreover, there is often no apparently optimal value of a goal, e. g. presumably more economic growth is always better than less.

(ii) A second possibility is to define all goals such that more is always better than less (e. g. more employment is always better than less) and to impose restrictions where necessary (e. g. employment cannot rise above 100 %). Though not all problems are evaded by this procedure, this approach will be used subsequently.

The resource limitations within which the government can operate are given by a production possibility set of the following general form

$$G[y_1, y_2 \ldots y_n; k(t), t] = 0, \qquad (10)$$

which is assumed strictly convex. $k(t)$ is a vector of exogenous factors entering non-randomly into the relationship among the goals. t indicates the possible shift of the relation over time which may be interpreted as the influence of technical progress.

The government does not directly control the goals, but must use instruments (such as monetary, fiscal or wage policy) for that purpose. As this paper concentrates on the political conflicts surrounding government behaviour, it is assumed that the government always has enough instruments available to control the goals within the resource limitations[19]. As is well known the dependence of goals upon instruments is empirically estimated in macro-econometric models.

[18] Such a loss function is used by Davis and Hinich (1967). For the use of a quadratic utility function see H. Theil (1961 and 1965).

[19] On the relationship between goals and instruments compare J. Tinbergen (1956). It is also assumed that the desired levels of the goals within the production possibility set can be instantly attained.

V. The Ideology Function and the Approval Function

A. Empirical Determination of the Ideology Function

Ideological preferences of the government and other parties are determined by the relative valuation attached to the different political issues[20]

$$I = I(y_1, y_2, \ldots y_n, t). \qquad (12)$$

It is assumed to be strictly concave with diminishing marginal utilities

$$\partial I/\partial y_i \gtrless 0 \qquad \partial^2 I/\partial y_i^2 \leqq 0 \qquad (i = 1, 2, \ldots n).$$

An ideology function would be a rather useless concept if it cannot be given any empirical content. In contrast to the general social welfare function widely used in economic theory and policy, the ideology function is amenable to empirical determination.

The ideology of a party is solemnly declared and constantly repeated in party programmes and manifestos. Content analysis of current speeches and writings by party leaders or special party ideologists provides further information. The construction of an ideology function may not always be easy as there are circumstances in which it is advantageous for a party to obscure it[21]. Nevertheless, there are definite features in each party's ideology which differentiates it from the other parties.

In the context of the traditional left-right spectrum of ideologies it is possible to establish a significant pattern of the relative ordering of goals. In Table 1 the ranking of economic goals from Socialists to Conservatives is shown for eight European countries combined.

The table shows that going from one extreme ideology to the other leads to an almost complete reversal of the relative importance given to the economic objectives. Such a table provides suggestive information on the ideology function but must not be interpreted as a cardinal ordering as in general there exist trade-offs between the different goals.

In the empirical determination of the ideology function care must be taken not to infer ideology simply from what ruling parties actually do. It is one of the contentions of this paper that the government may deliberately deviate in their actions from their most preferred ideological position. Information on ideology must hence be gathered independently of actual behaviour. However, it should be noted that given the present formulation of the problem, it is in theory possible to identify the weights used in the ideology function from government behaviour alone if the approval function and the economic system contraints are known *a priori* or have been estimated.

[20] Both ideological satisfaction and popular approval are defined such that they always lie between 0 and 1.

[21] Compare Downs (1957), chpt. 8.

Table 1.
Preferences of political parties with regard to economic goals 1964
(Synthesis for eight European countries)

Increasing Importance of Goals ↑	Party Ideologies		
	Socialists	Centre	Conservatives
	Full Employment	—	Price Stability
	Just Income Distribution	—	—
	—	Price Stability	—
	Satisfactory Growth	—	—
	—	Satisfactory Growth	Balance of Payments Equilibrium
	—	Full Employment	—
	—	Just Distribution	—
	Price Stability	—	Statisfactory Growth
	—	Balance of Payments Equilibrium	Full Employment
	—	—	—
	Balance of Payments Equilibrim	—	—
	—	—	Just Distribution

(Adapted from: Kirschen, Benard, Besters et al. (1964), Chapter IX, p. 227)

B. Empirical Determination of the Approval Function

It has already been pointed out that the popularity of the government as reflected e. g. in regular Gallup polls depends on how well it performs with respect to the various goals y. Without concerning ourselves with the problem of aggregating individual approvals into a social approval function, it is assumed that there exists a well-defined approval function which depends only on the levels of the various goals and possibly time. The approval function thus reads

$$A = A(y_1, y_2, \ldots y_n, t). \tag{11}$$

It can again be expected that there are diminishing marginal returns

$$\partial A/\partial y_i \gtrless 0; \quad \partial^2 A/\partial y_i^2 \lesseqgtr 0. \quad (i = 1, 2, \ldots n).$$

The approval function can be estimated empirically. Even the most casual observation of political life suggests that there is indeed quite a close interrelationship between economic conditions and government popularity. There are, however, only few studies concerned with this relation, and they focus mainly on the influence of the business cycle upon elections. A notable contribution is by L. H. Bean (1940 and 1948) who sees the business cycle as by far the most important predictor for the outcomes in both presidential and congressional elections in the United

States. He even derives an elasticity describing the average percentage swing against the ruling party when national income changes unfavourably. The famous Swedish economist J. Åkerman (1947) also finds the same kind of dependence for England, Germany and, less clearly so, for Sweden.

Modern statistical studies in the framework of a model — an approach which should be called "politometrics" — are almost non-existent, until recently. The most important contribution is by G. H. Kramer (1967). He explains the short-term variations in the share of votes received by the two major parties in United States elections. He succeeds in relating about half of the observed variations to the influence of economic factors. The explanatory variables are the growth rates of real and nominal income, the growth rate of prices (inflation) and the rate of unemployment. He finds that a higher income growth rate benefits the ruling party, an increase of inflation is favourable to the opposition while the influence of unemployment is not significant. The study clearly demonstrates a close relationship between government performance in fulfilling the goals and government popularity and that the approval function used in the present model has a counterpart in real life[22].

The politometric estimation of actual ballots can usefully be supplemented by sample surveys. In many countries such surveys are continuously undertaken and published on a journalistic level. Moreover, governments conduct (open or secret) surveys to find out the opinion of voters on certain issues, i. e., they in fact perceive the existence of an approval function.

There are also more refined scientific public opinion surveys in this area. Such studies are of particular interest because they can explore the voters' preferences through interlinking questions. An example has recently been published by Eva Mueller (1963) who establishes a rank ordering of public preferences with regard to government programmes which give useful indications about the approval function[23].

VI. The Formal Model and its Solution

Taking the various parts of the model together, the formal problem can be stated as follows[24]:

$$max\ U = \int_0^\infty F[I(y_1, y_2, \ldots y_n), A(y_1, y_2, \ldots y_n)] \cdot \alpha(t)\, dt, \tag{13}$$

[22] One must hasten to add that our concept of the approval function is an instantaneous one and depends only on instantaneous achievement of performance. Good-will, on the other hand, is formed by both past and present approval, appropriately depreciated. Election outcomes are determined by a weighted time average of good-will. Thus the approval function is more akin to a Gallup Poll rather than actual ballot behaviour.

[23] See also Campbell, Converse, Miller, Stokes (1960).

[24] Departing from the more general formulation, it is assumed here that the felicity, ideology and approval functions are autonomous, i. e. themselves independent of time.

subject to

$$G[y_1, y_2, \ldots y_n; k(t), t] \geqq 0, \tag{10}$$

and the transition equation

$$\dot{\pi}(t) = A(t) - \int_{-t_\infty}^{t} A(z)\, m(t-z)\, dz. \tag{4}$$

The condition whether the governments gets reelected or not has been expressed in the following form

$$\pi(NT) \geqq \overline{V}\left\{\frac{\pi(0)}{V(0)} + \int_0^{NT}\left[1 - \int_0^{t'+t_\infty} m(t'')\, dt''\right] dt'\right\} = \overline{\pi}(NT) \tag{9}$$

$$N = 1, 2, \ldots$$

To simplify the exposition, the term on the right hand side of (9) is denoted by $\overline{\pi}(NT)$.

The initial condition, stating that the government in power has been *elected* into office in the first place, is

$$\pi(0) \geqq \overline{V}\frac{\pi_0}{V_0}$$

or

$$V_0 \geqq \overline{V}.$$

It should be noted that the formal problem as set up is mathematically extremely complex. The special pecularities present in this political application are caused by the fact that the constraints are only relevant at discrete points of time, and that the state variable π appears only in the constaints. It is thus a *different class* of dynamic optimisation problems from that currently considered in optimal economic growth by K. J. Arrow and others[25]. For this reason it is not possible to apply directly either the classical calculus of variations or the Maximum Principle of L. S. Pontryagin and associates (1962); their approach requires a constraint set with an *interior* which does not exist in the present problem. In addition, it may be added that the transition Equ. (4) is generally an integro-differential equation and for arbitrary mortality density functions may not always be amenable to treatment by Pontryagin methods.

Several natural economic counterparts of the above optimisation problem immediately suggest themselves. They are:

(i) Optimisation problems associated with spatially or temporally fixed delivery quotas.

(ii) Optimisation problems of factory managers and administrators who are required to fulfil certain targets at specified points in time (this is probably especially relevant for state owned enterprises in socialist or communist economies).

(iii) Inventory problems.

[25] See e.g. Arrow (1967) and various articles collected by K. Shell (1967).

Despite the mathematical complexities, it is possible to characterise the solution to this optimisation problem. Yet in common with other optimisation problems involving inequality (or more frequently, non-negativity) constraints, it is not possible to give an explicit and analytical characterisation of the solution.

The ensuing discussion first establishes feasibility of the infinite horizon program, then deals with the effectiveness of constraints on reelection and finally gives a characterisation of the optimal policy, if one exists. Several specific examples will be given.

A. Feasibility

It must first be established that there exists *at least* one feasible programme, i. e. it must be possible to choose the goals $y_1(t), y_2(t), \ldots y_n(t)$ such that no constraints are violated at any time.

Since there exists no intertemporal relation among the y's in the model, it follows that any programme which maximises $A(t)$ at every point of time also maximises the good-will of the government $\pi(t)$ at every point. In particular, it maximises $\pi(t)$ at the nodal points (election dates) $t = NT$, where $N = 1, 2, \ldots$ (an integer). Thus feasibility is established if the path of $\pi(A_{\max})$ (with A maximised at every point) satisfies the constraints at the nodes, i. e. it is never lower than

$$\overline{\pi}(NT) = \overline{V}\left\{\frac{\pi(0)}{V(0)} + \int_0^{NT}\left[1 - \int_0^{t'+t_\infty} m(t'')\, dt''\right] dt'\right\} \tag{9}$$

at $t = NT$.

It is a specific characteristic of this political model that feasibility is *not* violated if $\pi(A_{\max})$ is below the limit $\overline{\pi}$ at points in *between* election dates. This path can be attained by maximising A with respect to the goals at every point in time

$$\max A(y_1, y_2, \ldots, y_n)$$

subject to:

$$G[y_1, y_2, \ldots y_n; k(t), t] \geqq 0.$$

Feasibility may be illustrated by the following figures.

It is seen that Fig. 1 pictures a feasible governmental programme because at the election dates the maximum good-will obtainable by disregarding all ideological considerations and maximising solely $A(t)$ is always higher than the limit $\overline{\pi}$, given by the points $A, B, C \ldots$ It is noted that during the terms (0 to T) and ($2T$ to $3T$) the government's good-will drops to below the limit, but this is without consequence.

The optimisation is non-feasible if at some NT the maximum possible good-will $\pi(A_{\max})$ accumulated by the government is insufficient for reelection (Fig. 2).

According to the figure, the government will not be reelected at the election held at $2T$, even if it tries everything to generate good-will

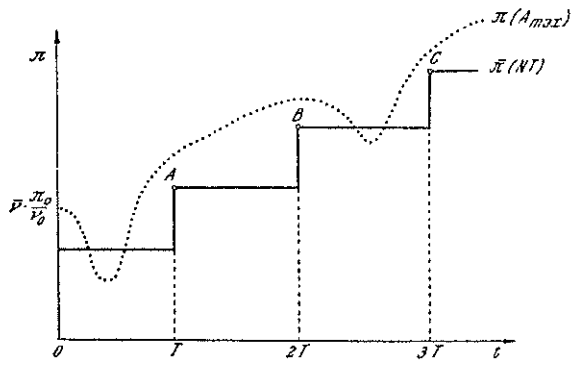

Fig. 1. Feasible path

amongst the voters. It must be noted that in the case of non-feasibility the optimisation problem as developed in this paper is changed: it is

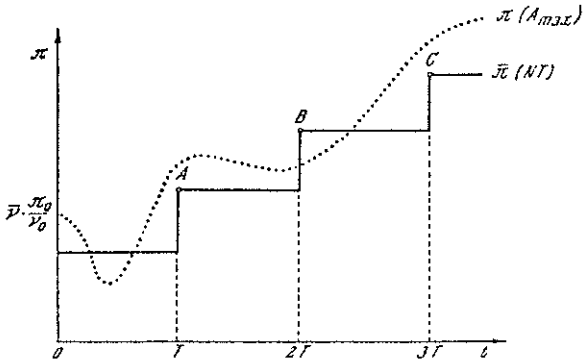

Fig. 2. A non-feasible path

necessary to substitute new assumptions about government behaviour. As this may lead too far, this line is not further explored here but only feasible programmes are considered[26].

[26] It can e. g. be assumed that in that case the government maximises its utility over the terms during which it is feasible to stay in office. Alternatively, the government may maximise its expected utility rather than utility. This will allow the government to go into opposition temporarily and regain power when circumstances become more favourable.

B. Effectiveness of Constraints on Reelection

Having assumed that there exists at least one feasible programme, it must be examined whether the contraints on reelection [Equ. (9)] are actually affecting government behaviour or not. It is certainly conceivable that a government whose ideology is very popular with the voters can safely disregard the election constraints, because it will be reelected with certainty.

Three possibilities can be envisaged:

(a) The Election Constraints are Never Binding

In that case the optimal government policy consists in the straightforward maximisation of its felicity at every point of time:

$$\max U = \int_0^\infty F\,[I\,(y_1, y_2, \ldots y_n),\ A\,(y_1, y_2, \ldots y_n),\ t]\ a\,(t)\ d\,t$$

$$= \max F, \text{ for every } t;$$

Subject to (10). The optimal path is (due to the concavity assumption imposed on F, A, I and G) well defined, if it exists (Fig. 3).

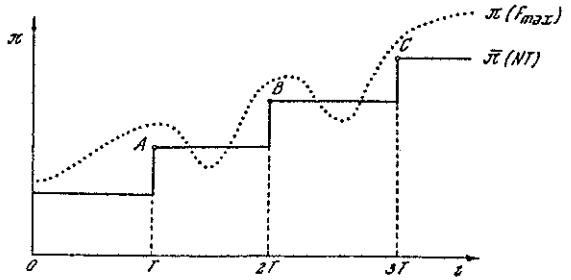

Fig. 3. No election constraint is binding

In fact, if a stationary environment prevails, i. e. $k\,(t)$ is a constant and G is independent of time, it can be shown that the optimal policy converges to a steady state path.

(b) All Election Constraints are Binding

This case is represented by Fig. 4. It can be seen that the government will lose each future election if it just maximises its felicity. It follows that such a policy violates the constraints imposed. In that case the optimisation problem can be broken up into *separate* parts. The government is confronted from each election to the next with the problem of maximising its utility over an interval of fixed lenght T in which the value of the state variable π is given for both the beginning and the end of the period. This is a well defined optimisation problem which can be solved with Pontryagin's Maximum Principle, or the Euler-Lagrange equation, after a transformation of the problem if necessary.

It should be emphasized that the fact that $\pi\ (F_{max})$ lies below the $\bar{\pi}$ (NT) for all N is not sufficient proof that all the constraints for reelection are binding. It is, however, very easy to find a felicity function for which

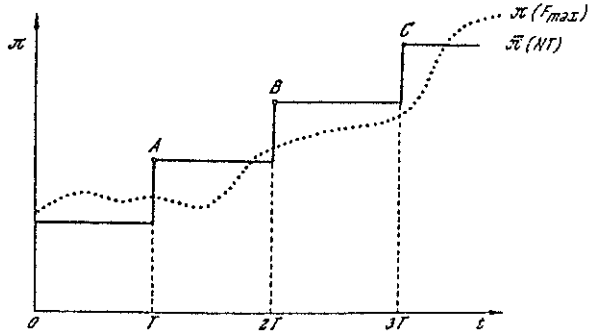

Fig. 4. All election constraints are binding

all the constraints for reelection are binding, e. g. if the ideological satisfaction function contains different arguments from the approval function and the felicity function assigns a small weight to approval. Under these conditions, fluctuations in policies may occur within each election interval, causing perhaps "business cycles" in the economy. The magnitude and phase of these fluctuations will of course depend very critically on the time structure of the mortality density function. Under the assumption of a stationary environment, without the complication of mathematics, the different types of fluctuations corresponding to different types of mortality density functions will be shown.

(a) Exponential deterioration. In this case, because of exponential deterioration of good-will, it is evident that one should postpone ad-

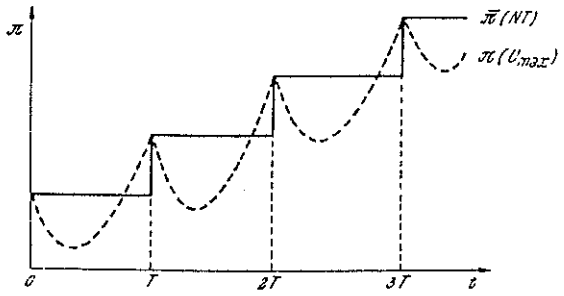

Fig. 5

ding to the good-will as much as possible, provided that it does not conflict with the felicity function. The optimum will look somewhat like the following (see Fig. 5).

It goes without saying that the instruments will undergo similar fluctuations.

(b) Sudden Death. In this case, memory is perfect for a finite period of time, after which everything is forgotten. If the constraints are

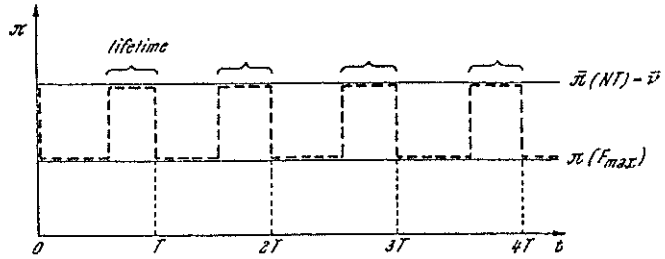

Fig. 6

to be binding for every election interval, it is clear that the finite period of the memory cannot be greater than T. This will lead to the following type of sawtooth fluctuations (Fig. 6).

(c) Impulse function. This corresponds to the case of no memory at all. The optimal policy is of course the policy given by the unconstrained maximisation of the discounted utility integral for all time, except for the election dates, at which time the optimal policy is given by setting

$$A\,(N\,T) = \bar{V}$$

excluding the case in which the unconstrained maximum is also the constrained maximum.

(c) Some Binding and Some Non-Binding Election Constraints

In general it must be assumed that the government is sometimes forced to depart from its felicity maximising policy in order to stay in office, while at other times it is not. Such a situation and its "backward" solution from $2\,T$ to 0, and then for the subperiods $2\,T$ to T and T to 0 is represented in Fig. 7.

A slight transformation of the problem is useful at this point. Let

$$\hat{\pi} = \pi - \bar{V}\left\{\frac{\pi_0}{V_0} + \int_0^t \left[1 - \int_{-t_\infty}^{t'} m\,(t'-t'')\,d\,t''\right] d\,t'\right\}. \tag{14}$$

Then

$$\begin{aligned}\dot{\hat{\pi}} &= \dot{\pi} - \bar{V}\left[1 - \int_{-t_\infty}^{t} m\,(t-t')\,d\,t'\right] \\ &= A\,(t) - \int_{-t_\infty}^{t} A\,(t')\,m\,(t-t')\,d\,t' - \bar{V}\left[1 - \int_{-t_\infty}^{t} m\,(t-t')\,d\,t'\right] \\ &= \left[A\,(t) - \bar{V}\right] - \int_{-t_\infty}^{t} \left[A\,(t') - \bar{V}\right] m\,(t-t')\,d\,t'. \end{aligned} \tag{15}$$

Under this transformation, $\hat{\pi}$ becomes the state variable and (15) becomes the transition equation.

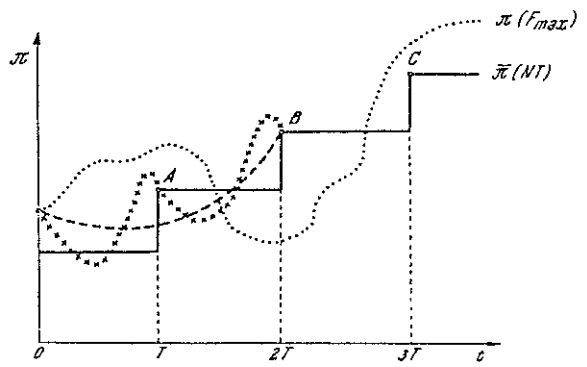

Fig. 7. Coexistence of binding and non-binding election constraints

The problem then becomes

$$\underset{y_t}{\text{Max}}\ U = \int_0^\infty F(t)\, a(t)\, dt$$

subject to

$$G[y_1, y_2, \ldots, y_m, k(t), t] = 0. \tag{i}$$

$$\dot{\hat{\pi}} = [A(t) - \bar{V}] - \int_{-t_\infty}^{t} [A(t') - \bar{V}]\, m(t - t')\, dt', \tag{ii}$$

$$\hat{\pi}_0 = \pi_0 \left(1 - \frac{\bar{V}}{V_0}\right), \tag{iii}$$

$$\hat{\pi}(NT) \geqq 0,\ N = 1, 2, \ldots \text{ an integer.} \tag{iv}$$

It is immediately evident that if $\bar{V}/V_0 < 1$, then as long as $\dot{\hat{\pi}} \geqq 0$, feasibility is assupped. Moreover, we note that $\bar{V}$ is typically of the magnitude of fifty percent. If under a stationary environment, $\max A \geqq \bar{V}$, it is seen that there always exists at least one feasible programme.

If it is assumed that the constraint (iv) is binding at every instant rather than at discrete points in time, then the optimal policy, if one exists, will consist of alternations of free (non-negativity constraints not binding) and blocked (non-negativity constraints binding) intervals, using the terminology introduced by Arrow (1966). In a free interval, the

path will follow a felicity maximising rule, which may be described as "myopic". In a blocked interval, a constant zero level of $\hat{\pi}$ will be maintained.

It is also evident, however, that the above solution to the optimisation problem is not in general the optimum, as a more stringent constraint is imposed. (iv) is assumed to hold at all instants of time instead of at discrete points. One expects the value of the utility integral to be increased (or at least not decreased) by imposing the non-negativity constraints only at $t = NT$.

The solution to the optimisation problem imposing the constraints at discrete points in time is complex. However, it is possible to state the following characterisation of the optimal solution:

Proposition I. The optimal solution, unless it also coincides with the unconstrained maximum of the utility integral, must have the property that $\hat{\pi}(NT) = 0$ for some N. Thus, one can restrict the search for an optimum solution from amongst the class of feasible solutions exhibiting this particular property. This property of the time path of $\hat{\pi}$ will be known as the *critical point property*[27].

Proposition II. A feasible path with the critical point property is said to be *dominated* by another feasible path with the critical point property if the two paths have identical critical points and the latter path yields a higher value of the utility integral. The optimal solution to the problem must therefore be an undominated feasible path with the critical point property. It is evident that given all the critical points, a sub-optimisation problem may be solved to determine the optimal path passing through all these critical points. A piecewise application of either Pontryagin's maximum principle or the Euler-Lagrange equation between adjacent critical points will yield the undominated path given the set of critical points (as e. g. between 0 and T, and T to $2T$ in Fig. 7).

Proposition III. A solution is termed *admissible* if it is a feasible and undominated path with the critical point property *and* if there does not exist another feasible and undominated path with the critical point property whose critical points are completely contained in the set of critical points of the former. The optimal solution must be an admissible solution.

It is evident from the above propositions that there exists only one undominated strategy for a given set of critical points (which is not necessarily feasible). It is also evident that intersection of the critical points corresponding to each admissible solution must be non-empty. In particular, the intersection of the critical points of *any* two admissible solutions must be non-empty.

Thus, for the purpose of finding an optimal solution, one needs only consider the class of admissible solutions. One can compute the values of the utility integral corresponding to each admissible solution and then choose the one with the highest value. This must then be the optimal solution.

[27] See Manne (1968) for an excellent discussion of some similar problems.

For the finite horizon case a dynamic programming type technique may be employed to search from amongst the class of undominated solutions the admissible solutions without the assumption of stationarity and/or stationary environment. For the case of infinite horizon, no algorithms have been found that solve the problem in a non-stationary environment. However, under the assumption of stationary environment, a recursive method of solution appears feasible[28].

VII. Some Concluding Remarks

As emphasised at the beginning of the paper, a theory of the behaviour of the government is useful and indeed indispensable for a great many purposes. It is not possible here to show how the theory developed will bear on these problems or to explore in detail the working of the model. It must suffice to indicate sketchily how the model can be employed to enlighten one specific topic as an illustration, namely how the political system is able to generate fluctuations in economic activity ("Business Cycles"). At the same time this topic serves to bring out more clearly some of the properties of the model.

Many students of business cycles have noted perhaps casually that a large share of the cycles can or should be attributed to political causes[29]. Though this sounds intuitively obvious, it is nevertheless difficult to see how the political system (and more specifically the government) can generate such cycles in either absolute magnitudes, or, what is more relevant in a modern economy, in their rates of change. Neither one of the theories of government behaviour advanced so far (i. e. the maximisation of ideological satisfaction or vote maximisation) seems to be able to explain the creation of such fluctuations.

The model of government behaviour developed here lends itself to an analysis and explanation of such politically caused cycles in economic activity. As the economic system forms an integral part of the model, the translation of political forces into economic actions, and conversely, can be studied in explicit form.

The model permits one to draw the following inferences[30].

(a) As shown above, it is *logically possible* that the political system produces economic cycles. In fact, it is extremely likely in countries where there is no dominant ideology subscribed to by a large share of the population. The fluctuations in the economic variables will be of an approximately regular length, corresponding to the institutionally fixed election periods. Under the stationarity assumption, the form of the cycle will only be exactly repeated if the exogenous factors of the model are

[28] The mathematics will be explored more formally in another paper.

[29] E. g. Schumpeter (1935) who holds that "...we could without any glaring absurdity account by political "disturbances" for all the fluctuations we observe in our material" (p. 2).

[30] Space and time limitations do not permit an exhaustive or rigorous analysis here. These will be pursued in greater detail elsewhere.

unchanged *and* if random disturbances are absent. The model is also capable of showing how e. g. a gradual, but continuous worsening of general economic conditions or an adverse random disturbance may influence the pattern of the cycle. Also according to the model, the tendency for politically induced fluctuations in economic activity will *cet. par.* increase as election approaches because the government will be under pressure to depart more and more from its utility maximising policy as each election approaches. This suggests that (at least on this account) a decreasing growth rate of the economy, for instance, is likely to be accompanied by more intense cycles.

(b) It is also evident that the less popular the government, the larger are *cet. par.* the politically generated fluctuations. When a party in power has a large majority and confidently expects to retain it over future terms, it is not forced to change its policy before elections in order to accumulate sufficient good-will with the voters for reelection.

A government may be popular for two different reasons:

(i) The general economic conditions may be favourable to the government [as reflected by $k(t)$ in Equ. (10)], e. g. because the country is provided with much infrastructure. The voters are then satisfied with the government, as it is able to *simultaneously* present more favourable positions on *all* goals[31].

(ii) There is little difference between the government's ideological preferences and the voters' approval function. In that case, the weights of the arguments in the ideology and approval functions are similar. The government can then pursue a felicity maximising strategy (subject, of course, to feasibility).

(c) The politically generated cycle *cet. par.* gains momentum towards the election date because the government knows that accumulated goodwill tends to depreciate. The exact time path crucially depends on how the instruments at the government's disposal influence the time path of the goal variables[32].

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[31] It has been suggested that such a situation was present in the German Federal Republic in the Fifties and early Sixties. Compare B. Frey (1968).

[32] It should be noted that throughout this analysis it is assumed that the instruments chosen affect the goals instantaneously. In general, there will be a lagged response structure.

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